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Solving Bi-Objective Fuzzy P-Hub Center Problem by Using Genetic Algorithm, Incorporating Local Search and Absolute Priority Method

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Abstract

Fuzzy p-hub center problem is proposed and solved by genetic algorithm incorporating local search (GALS) before. In real world, opening hubs may need considerable cost. In this situation, minimization of the hub numbers is important, too. So in this paper, a bi-objective p-hub center problem is presented. To solve the proposed bi-objective model, we suggested an algorithm using GALS based on absolute priority of objectives. Optimum numbers of hub are calculated in two numerical examples by proposed algorithm.

Keywords: Bi-Objective Fuzzy P-Hub Center Problem, Genetic Algorithm Incorporating Local Search (GALS), Bi-Objective Mixed-Integer Linear Programme

Introduction

One of the location-allocation models is max-min distance model, which is usually applied for the issue of location and allocation of emergency service centers, distributors of perishable goods, and express post. In this model, hubs are central facilities and another points should be connect through hubs.

Fuzzy p-hub center location problem is a hybrid optimization problem, in which p number of hubs is selected in the network (a set of n predetermined locations). Then, Spokes (the nodes which are not hubs) are allocated to the hubs, so maximum travel time (cost, distance, etc.) between each source and destination is minimized, while fuzzy variable of travel time is trapezoidal (Yang et al, 2013).

In general, the research in the field of hub location can be classified into three categories:

1. Simplifying mathematical models and presenting a better model with less variables and constraints
2. Changing the model and making it applicable
3. Presenting better solutions

Studies in recent years have been more focused on solution methods, i.e. the third category. However, this article focused on second category and an applicable bi-objective fuzzy p-hub center location is presented.

Campbell (2002) and O'Kelly and Miller (1994) have classified different kinds of hub location models. This classification was continued by Farahani et al (20134) from 2007 to 2013.

P-hub center location problem in discrete space was first proposed by Campbell (1994) and basic assumptions were established in his model. He also formulated a quadratic zero-one integer programming model. Kara and Tansel (2000) proved that p-hub center location problem is NP-hard and presented several linear programming models considering Campbell's model. Additionally, they proposed a new model and solved it using precise methods and showed the new model is better than the Campbell model according to computation time of solving the problem.

Ernst et al (2007) presented a mixed integer linear programming based on the concept of radius of hubs for single and multiple allocation of uncapacitated p-hub center location problem. In presented model, the p-hub center location problem was a subset of the main problem. They proved that the single allocation problem is NP-hard and presented branch-and-bound solution for solving multiple allocation. The calculation results shows that their solution were better than Kara and Tansel's solution (2000). However, while their proposed method works well in small size cases, but it was weak in large size ones.

Campbell et al (2007) examined a subset of p-hub center location problems, in which location of hubs was predetermined. They presented an integer programming formulation for single and multiple allocation problems in two capacitated and uncapacitated models and calculated degree of difficulty for each one. They also found that most of the problem models were NP-hard. They examined a special network with two hubs and presented a polynomial time algorithm to solve it. Their proposed algorithm works well in a certain network of points, while innovative algorithms are needed for finding solutions to common models.

Innovative algorithms were first applied by Pamuk and Sepil (2001). They presented single allocation innovative algorithm along Tabu search for solving p-hub center location problem; this algorithm can solved large-sized numerical problems in acceptable time.

Yaman and Elloumi (2012) presented star p-hub center problem with single allocation. This model was different, since some node(s) were previously determined as stars and directly connected to the hubs. All other non-hub nodes were allocated to the hubs so that each non-hub node would be only allocated to only one hub. Also, Liang (2013) presented the NP-hard type of the model and the problem's solution.

In hub and Spoke systems, there are a large number of uncertain factors, which affect location decision making process (e.g. demands, costs, time, and other parameters) that can influence the decision-making of spatial processes such as demands, price, time, and other parameters. The importance of this uncertainty has led many researchers to consider stochastic and uncertain parameters in p-hub center location problem. For example, Yang et al (2013) solved p-hub center location problem in fuzzy environment (via the fuzzification of travel time) using GALS method, their solution was considerably better than LINGO method and standard genetic algorithm in speed and accuracy. In that model, travel time was trapezoidal fuzzy and normal fuzzy variable. Yang et al. considered travel time as the second-type fuzzy trapezoidal variable and presented parallel mixed integer linear programming model that can be solved using general optimization software. They run their model on the network with 15 nodes, the results showed the acceptable accuracy and efficiency of the solution.

Opening hubs may need noticeable cost in real world (e.g. storage of goods, post offices, and emergency centers). So, minimization of the hub numbers is important, too. According to the recent investigations, finding optimal number of hubs has not been studied. To find the optimal number of hubs, a bi-objective model is proposed. This model was based on p-hub center location model presented by Yang et al (2013) with fuzzy trapezoidal travel time variables. Also, GALS was used for solving the model.

In Section 2, bi-objective fuzzy p-hub center location model is presented, in Section 3, GALS is described, and numerical calculations are presented in Section 4. Finally, conclusions are presented in Section 5.

Bi-objective fuzzy p-hub center model

In some transportation networks such as emergency service systems, facility installation in hubs needs noticeable cost. Hence, in this paper, a modification of p-hub center location problem would be presented to find the optimum number of hubs. To achieve this, the second objective function was added to fuzzy p-hub center location model which want minimizing the number of hubs. In this bi-objective model, the first priority is minimizing maximum travel time (like regular fuzzy p-hub center problem). When the first objective was not met, the objective of minimizing p value (second priority) could not be provided (absolute priorities method). To solve the model, fuzzy p-hub center location problem was solved with a certain confidence degree, size, and discount coefficient and different numbers of hubs. Then the value of the objective function were compared. With increasing p value, where the value of objective function was not improved, the minimum required number of p was determined. In the proposed model, variables, indices, and parameters were according to the model presented by Yang et al (2013).

$$\text{Minimize} \quad Cost \tag{1}$$

$$\text{Minimize} \quad Z \tag{2}$$

$$s.t. \quad Cr\{(T_{ik} + dT_{km} + T_{mj})X_{ikmj} \leq Z\} \geq \alpha \quad \forall i,k,m,j \in N \tag{3}$$

$$X_{ikmj} \geq X_{ik} + X_{mj} - 1 \quad \forall i,k,m,j \in N \tag{4}$$

$$\sum_{k \in N} X_{ik} = 1 \quad \forall i \in N \tag{5}$$

$$X_{ik} \leq X_{kk} \quad \forall i,k \in N \tag{6}$$

$$\sum_{k \in N} X_{kk} = p \tag{7}$$

$$X_{ik} \in \{0,1\} \quad \forall i,k \in N \tag{8}$$

$$X_{ikmj} \in \{0,1\} \quad \forall i,k,m,j \in N. \tag{9}$$

$$cost = Dp \qquad p = 2, \dots, n \tag{10}$$

In models (1)-(10), objective function (1) is to minimize the number of hubs in the second priority, and (2) relates to minimize the value of fuzzy objective function with confidence degree α in the first priority. Confidence degree α is a value between zero and one. To minimize the maximum travel time with confidence degree α, constraint (3) with objective function (2) is used. In constraint (4), if path i → k→ m → j exists in the network, it means that nodes i and j are connected to hubs k and m, respectively. In constraint (5), each node is assigned to only one hub. In constraint (6), nodes can be only allocated to hubs. Exactly p hubs exist in the problem which is determined in constraint (7). Constraints (8) and (9) show zero-one variables. In constraint (10), value D is spent on installing the facility in a hub.

In the above mathematical programming model, trapezoidal fuzzy travel times are assumed as $T_{ik} = (r_{ik}^1, r_{ik}^2, r_{ik}^3, r_{ik}^4), T_{km} = (r_{km}^1, r_{km}^2, r_{km}^3, r_{km}^4), and\ T_{mj} = (r_{mj}^1, r_{mj}^2, r_{mj}^3, r_{mj}^4)$, which are independent by two .For constraint (3) to be linear, considering objective (2) of the problem, two states of smaller and larger than α are considered. In each state, by linear combination of times and α along with defuzzification of times, a coefficient is made for zero-one Xikmj variable. Hence, the constraint related to determining validity degree is formulated as Relation (11) .

$$f(X_{ikmj}) \leq Z, \quad \forall i,k,m,j \in N \tag{11}$$

Where $f(X_{ikmj})$ can be calculated using relation (12).

$$f(X_{ikmj}) = \begin{cases} \left((1-2\alpha)(r_{ik}^1 + dr_{km}^1 + r_{mj}^1) + 2\alpha(r_{ik}^2 + dr_{km}^2 + r_{mj}^2)\right) X_{ikmj} & \alpha \leq \frac{1}{2} \\ \left((2-2\alpha)(r_{ik}^3 + dr_{km}^3 + r_{mj}^3) + (2\alpha-1)(r_{ik}^4 + dr_{km}^4 + r_{mj}^4)\right) X_{ikmj}, & \alpha > \frac{1}{2} \end{cases} \tag{12}$$

Proof of relation (12) is available in [1].

Proposed algorithm

GALS for solving fuzzy p-hub center location problem was proposed by Yang et al (2013). Genetic algorithm, as a popular method in solving difficult optimization problems, was presented by Holland (2014). This algorithm is a stochastic search process, which is designed based on natural selection mechanisms, genetics, and evolution. On the other hand, local search is an applicable and general method for finding near-optimal solutions in difficult optimization problems. This method starts with an initial solution and searches for neighboring or local solutions.

GALS is a hybrid algorithm in which genetic algorithm plays the key role and local search is considered as an intelligent mutation. GALS utilizes local search method at the beginning, which has better performance than standard genetic algorithm. When the algorithm starts with a good solution, it is usually more efficient in finding the optimal solution with less repetition. Yang et al (2013) showed that for solving fuzzy p-hub center location problem, GALS is better than genetic algorithm.

In proposed algorithm, GALS may be applied several times to find p*. Steps of proposed algorithm are as follows:

Step 0: Specifying initial population size, mutation rate 1, mutation rate 2, and rate of crossover operation. Also consider p=2.

Step 1: Stochastically generating initial population or initial chromosomes and applying local search to the generated chromosomes.

Step 2: Calculating fitness of each chromosome by objective function value and selecting chromosomes by Roulette Wheel.

Step 3: Updating the chromosome by cross process and improving the offspring using local search.

Step 4: Updating the chromosome by mutations and improving the offspring using local search.

Step 5: Performing steps 2 to 4 with a certain number of repetition.

Step 6: Reporting the best chromosome as the optimal solution.

Step 7: Save p value and optimal objective function value then consider p=p+1.

Step 8: If generating initial population is possible, go to step 1.

Step 9: Reporting the least p which has the least objective function value as p*.

Results

To solve the proposed model, the data set including 25 nodes was used (Figure 1). These nodes were stochastically generated within [0,100] and were classified as sizes 15 and 25 for problem solving.

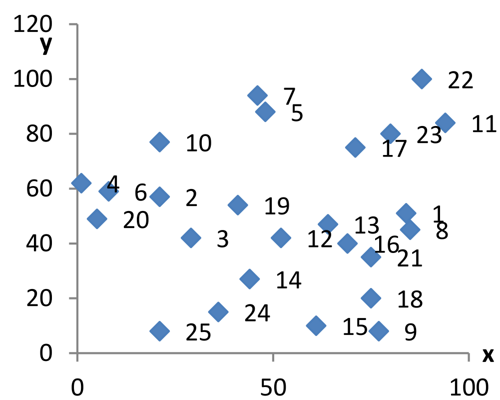

Figure 1: Location of 25 nodes in the coordinate system

Parameters of discount coefficient and confidence level were considered 0.6 and 0.9, respectively. GALS was coded in MATLAB and set with the primary population size of 10, repetition number of 6, probability of intersection or cross operations of 3.0, and probability of change type mutations in chromosomes 0.2 and 5.0.

Fuzzy p-hub center location problem using GALS was done with the aim of finding the optimal p value with the sizes of 15 and 25 nodes and the results are given in Table (1).

Table 1. Results of solving fuzzy p-hub center location problem using GALS with discount coefficient 0.6 and confidence level 0.9

p	Second objective function value	
	n=15	n=25
2	39.24	48.24
3	39.24	45.24
4	35.24	45.24
5	35.24	40.64
6	30.64	30.64*
7	30.64	30.64
8	25.24*	30.64
9	25.24	30.64
10	25.24	30.64
11	25.24	30.64
12	-	30.64
13	-	30.64
14	-	30.64
15	-	30.64
p*	8	6

According to the results of solving the problem with two sizes of 15 and 25 nodes, it is clear that the optimal value of the objective function was improved by increasing the number of hubs. However, such improvement stopped somewhere, which indicated the optimal number of hubs. For the mentioned problem with two sizes of 15 and 25 nodes, if the installation cost of each facility was one unit, the best problem response would be obtained with 6 and 8 hubs, respectively.

Conclusion

Many solution methods and models have been presented for p-hub center location problem, before. However, none of them has investigated the optimum number of p. Since it may be needs considerable cost to install facilitators in hubs, the present study for the first time tried to model Fuzzy p-hub center problem with the ability of determining the optimum p value. In this bi-objective model, the first priority was to minimize maximum travel time and the second was to minimize installation cost of opening the hubs. The proposed model was solved using GALS, the results were presented, and the optimum number of p was calculated.

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